

# Risk-Aware Multi-Robot Target Tracking with Dangerous Zones

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**Abstract**—Multi-robot target tracking is the problem of actively planning the motion of a team of coordinating robots such that they can track and follow the targets. Traditional approaches often focus solely on optimizing the tracking accuracy, neglecting potential environmental hazards that can induce system faults such as sensor or communication failures on the robots. In this paper, we focus on solving multi-robot target tracking in an adversarial environment with two types of dangerous zones—sensing danger zone and communication danger zone. Robots’ sensors are at the risk of being damaged if robots are in the sensing danger zone, while inter-robot communication signals can be jammed if robots are in the communication danger zone. The positions of both types of zones are uncertain and assumed to follow a Gaussian distribution. We model the risk of attack in dangerous zones as probabilistic constraints, and propose an optimization program that optimizes the tracking performance while ensuring robots’ safety through such constraints. The probabilistic constraints are linearly approximated so that the optimization program becomes tractable. We demonstrate the efficacy of the proposed approach through extensive simulations.

## I. INTRODUCTION

An increasing amount of research effort has been dedicated to multi-robot active target tracking, which refers to the problem of planning the (joint) motion of a team of robots such that they could optimize certain tracking objectives. However, the practical deployment of multi-robot target tracking remains challenging, as the environment or targets themselves can be dangerous or adversarial. Robots may be subject to attacks resulting in sensor damage, communication interruption, or other system faults. Therefore, tracking performance is no longer the only objective to achieve, as the robots must also pay special attention to secure themselves from failures or attacks. Previous work [1]–[4] closely studies multi-robot target tracking under the adversarial setting. Although awareness of risk and resiliency have been investigated in these works, the worst-case assumption is often adopted, leading to an excessively conservative decision-making paradigm. In this paper, we alternatively consider risk in a probabilistic setting and model the safety requirement on robots, *i.e.* the risk level must not exceed a threshold, using chance-based constraints.

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These chance-based constraints do not have closed forms in general and are thus challenging to compute directly. We obtain inspiration from how motion planning algorithms deal with uncertainty [5, 6], and apply approximation techniques when evaluating such probabilistic constraints to alleviate the computational difficulty.

Our major contributions are:

- We model the problem of multi-robot active target tracking with the existence of uncertain dangerous zones as a novel chance-constrained optimization.
- We approximate the probabilistic constraints to make it computationally tractable and simplify the optimization program.
- The proposed method is then extensively verified through simulations.

## II. PROBLEM FORMULATION

We consider the active target tracking problem with  $M$  robots and  $N$  targets in the environment with two types of dangerous zones, where each type of zone performs one kind of adversarial attack on the robots. We aim to enable the robots to minimize their estimation uncertainty of the target positions, while simultaneously securing themselves from attacks. In the upcoming sections, we first introduce the definitions of dangerous zones and then formulate the multi-robot target tracking as a chance-constrained optimization problem. For the rest of this paper, we will use  $[N]$  to denote the set  $\{1, 2, \dots, N\}$  for an arbitrary  $N \in \mathbb{N}^+$ .

### A. Sensing danger zone

We assume that there are  $p$  disk-shaped sensing danger zones in the workspace, denoted as  $\{\mathcal{S}_1, \dots, \mathcal{S}_l, \dots, \mathcal{S}_p\}$ . At the center of each sensing zone, there exists a stationary sensor from the enemy. Whenever a robot enters a sensing zone, it will be detected and its own sensor suite is subject to attacks. To protect our robots, we want to restrict the probability that the robots enter sensing danger zones.

We assume sensing danger zones to be static and uncertain. Specifically, the hostile sensor zone  $\mathcal{S}_l$  is defined with a center position  $\mathbf{x}_{\mathcal{S}_l} \in \mathbb{R}^2$  and boundary radius  $r_l$ , where  $\mathbf{x}_{\mathcal{S}_l}$  follows a known Gaussian distribution  $\mathcal{N}(\mu_{\mathcal{S}_l}, \Sigma_{\mathcal{S}_l})$ . For the  $i$ -th robot at position  $\mathbf{x}_i$ , the probability of entering any sensing zone and being detected should be upper bounded by a chance constraint

$$\text{Prob}(\mathbf{x}_i \in \mathcal{S}_l) \leq \epsilon_1, \forall i, \forall l, \quad (1)$$

where  $\epsilon_1$  is a pre-specified parameter. Equivalently, robot is within the danger zone when the distance between it and the hostile sensor is no greater than radius of the zone,

i.e.,  $\|\mathbf{x}_{S_l} - \mathbf{x}_i\| \leq r_l$ . Hence, the probability in Eq. 1 can be computed as:

$$\begin{aligned} \text{Prob}(\mathbf{x}_i \in \mathcal{S}_l) &= \int_{\|\mathbf{x}_{S_l} - \mathbf{x}_i\| \leq r_l} \text{pdf}(\mathbf{x}_{S_l} - \mathbf{x}_i) d(\mathbf{x}_{S_l} - \mathbf{x}_i), \\ &= \int_{\|\mathbf{w}_{i,S_l}\| \leq r_l} \text{pdf}(\mathbf{w}_{i,S_l}) d\mathbf{w}_{i,S_l}, \end{aligned} \quad (2)$$

where  $\mathbf{w}_{i,S_l} = \mathbf{x}_{S_l} - \mathbf{x}_i$ . The probability is an integral of a multivariate Gaussian random variable without an explicit closed form. To address this issue, we will linearize the region  $\{\mathbf{w} \mid \|\mathbf{w}_{i,S_l}\| \leq r_l\}$  and compute an upper bound of the probability in Eq. 2. The details are introduced in Section III.

### B. Communication danger zone

Communication danger zones are the ones where “jamming” happens. We assume all communication danger zones to be disks and stationary throughout the whole process. There exists a hostile station at the center of each communication danger zone. If a robot enters a communication danger zone, the station can send noisy, deceiving signals to jam the robot, such that the robot fails to distinguish between the message from its neighboring teammates and the jamming signals. Since jamming reduces the reliability of inter-robot communication, we consider how to reconfigure the robots’ positions to discourage jamming effect.

Consider there are  $q$  communication danger zones as  $\{\mathcal{C}_1, \dots, \mathcal{C}_k, \dots, \mathcal{C}_q\}$  in the environment. We use  $\mathbf{x}_{C_k} \in \mathbb{R}^2$ ,  $k \in [q]$  to represent the position of the jamming station in communication danger zone  $\mathcal{C}_k$ . Similar to sensing danger zones, we also assume  $\mathbf{x}_{C_k}$  is a random variable following a known Gaussian distribution,  $\mathbf{x}_{C_k} \sim \mathcal{N}(\mu_{C_k}, \Sigma_{C_k})$ ,  $\forall k$ .  $\psi_i$  denotes the set of neighbors of robot  $R_i$ , which are basically teammates within a prespecified communication range.

Without loss of generality, we consider a robot  $i$  inside a communication danger zone  $\mathcal{C}_k$ . The jamming station can attack the communication channel between robot  $i$  and its neighbors  $j \in \psi_i$ . Let the distance between robot  $i$  and  $j$  be  $c$ , and the distance between robot  $i$  and the jamming station of  $\mathcal{C}_k$  be  $a \in \mathbb{R}$ . We assume a lower-level controller guarantees robot would not collide with its teammates, and thus  $c \in \mathbb{R}^+$ . We encode the chance that robot  $i$  successfully receives the messages passed from its neighbor  $j$ , even though under jamming attack, using a metric called “Signal-to-Jamming Ratio”, defined as

$$\gamma_{ijk} = \frac{a}{c}, \quad (3)$$

and we require  $\gamma_{ijk} \geq \delta_2$ , where  $\delta_2$  is a hyper parameter. To increase the chance that robot  $i$  reliably receives information from  $j$ , it is desirable for robot  $i$  to be closer to robot  $j$  and away from the jamming station at  $\mathbf{x}_{C_k}$ . Therefore, the ratio between  $a$  and  $c$  should be lower bounded. We employ a chance-based constraint to ensure robots’ safety in communication danger zones:

$$\begin{aligned} \text{Prob}\left(\frac{a}{c} \geq \delta_2\right) &\geq \epsilon_2, \\ \iff \text{Prob}\left(\frac{a}{c} < \delta_2\right) &\leq 1 - \epsilon_2. \end{aligned} \quad (4)$$

Let  $\mathbf{x}_i$ ,  $\mathbf{x}_j$ ,  $\mathbf{x}_{C_k} \in \mathbb{R}^2$  denote the positions of robot  $i$ , robot  $j$ , and the jamming station in  $\mathcal{C}_k$ , respectively. The probability in Eq. 4 can be further computed as

$$\begin{aligned} \text{Prob}\left(\frac{a}{c} < \delta_2\right) &= \text{Prob}(a < \delta_2 c), \\ &= \int_{\|\mathbf{x}_{C_k} - \mathbf{x}_i\| < \delta_2 c} \text{pdf}(\mathbf{x}_{C_k} - \mathbf{x}_i) d(\mathbf{x}_{C_k} - \mathbf{x}_i), \\ &= \int_{\|\mathbf{v}_{i,C_k}\| < \delta_2 c} \text{pdf}(\mathbf{v}_{i,C_k}) d\mathbf{v}_{i,C_k}, \end{aligned} \quad (5)$$

where  $\mathbf{v}_{i,C_k} = \mathbf{x}_{C_k} - \mathbf{x}_i$ . Note that Eq. 5 integrates the probability density function of a multivariate Gaussian variable  $\mathbf{v}_{i,C_k}$  across a disk that is centered at the origin and has radius  $\delta_2 c$ . For robot  $i$ , each of its neighbors will form a corresponding constraint. We define  $c^* = \max\{c_j \mid c_j = \|\mathbf{x}_i - \mathbf{x}_j\|, j \in \phi_i\}$ , which corresponds to the radius of the largest circle across those we perform the integration in Eq. 5 for robot  $i$  on. To protect robot  $i$  from jamming attack in dangerous zone  $\mathcal{C}_k$ , we require

$$\text{Prob}(a < \delta_2 c) \leq \text{Prob}(a < \delta_2 c^*) \leq 1 - \epsilon_2. \quad (6)$$

Directly computing Eq. 6 is challenging and we will further explain the approximation approach in Section III.

### C. The chance-constrained optimization program

We present a chance-constrained optimization program in Eq. 7 to solve the optimal actions of all the robots at every time step  $t$ . The objective function is a weighted sum of tracking uncertainty, as encoded by the trace of the estimation covariance matrix, and control efforts of the whole team. Weights for the two pieces are  $w_1$  and  $w_2$ , respectively. Chance-based constraints are imposed to restrict the probability that robots are attacked in sensing and/or communication danger zones.

$$\min_{\mathbf{u}_{i,t}, \mathbf{x}_{i,t+1}, \forall i} w_1 \cdot \text{Tr}(\mathbf{P}_{t+1}) + w_2 \cdot \sum_{i=1}^m \|\mathbf{u}_{i,t}\| \quad (7a)$$

$$\text{s.t. } \mathbf{x}_{i,t+1} = \mathbf{f}_i(\mathbf{x}_{i,t}, \mathbf{u}_{i,t}), \forall i \in [M], \quad (7b)$$

$$\text{Prob}(\|\mathbf{x}_{S_l} - \mathbf{x}_{i,t+1}\| \leq r_l) \leq \epsilon_1, \forall i \in [M], \forall l \in [p], \quad (7c)$$

$$\text{Prob}(a_{ik} < \delta_2 c_i^*) \leq 1 - \epsilon_2, \forall i \in [M], \forall k \in [q]. \quad (7d)$$

Eq. 7c is the constraint preventing robots from entering the sensing danger zone. Eq. 7d is the constraint to preserve effective inter-robot communication in communication danger zones. We use  $a_{ik}$  to represent the distance between robot  $i$  and the jammer of communication danger zone  $\mathcal{C}_k$ , and  $c_i^*$  to represent  $c^*$  value of robot  $i$ , both calculated using robot  $i$ ’s position at time step  $t + 1$ . Eq. 7b is the dynamics constraint for each robot. In this work, we use single-integrator dynamics for all robots.

Optimization in Eq. 7 is computationally challenging to solve, since it computes trace of the covariance matrix  $\mathbf{P}_{t+1}$  at time step  $t + 1$  in the objective function. This requires propagating an Extended Kalman Filter (EKF) for

one step. The measurement noise covariance matrix we use in the update step of EKF is a nonlinear function of the distance between robots and targets, *i.e.* approaching the targets reduces noise level. As such, the objective function is non-convex. The probabilities in Eq. 7c and Eq. 7d are also nonconvex and cannot be obtained directly. We introduce our method of simplifying this optimization problem in the next section. Note that whenever the robots move to a new state, they run an EKF to update their estimates of the target positions.

### III. APPROACH: APPROXIMATE CHANCE CONSTRAINTS

The probability in both Eq. 2 and Eq. 5 requires computing the integral of multivariate Gaussian variables across disk regions, which cannot be done easily with a closed form. To resolve this issue, we approximate the probabilities using an upper bound and transform the chance-based constraints into deterministic ones.

We firstly review the following lemmas in [5, 6]. Let  $\mathbf{x} \in \mathbb{R}^{n_x}$  be a random variable that has a Gaussian distribution. We consider a general probabilistic constraint of the form,  $\text{Prob}(\mathbf{a}^\top \mathbf{x} \leq b) \leq \delta$ , where  $\mathbf{a} \in \mathbb{R}^{n_x}$ ,  $b \in \mathbb{R}$  are constants, and  $\delta$  is the confidence level.

*Lemma 1:* Under the assumption that  $\mathbf{x}$  is a multivariate Gaussian random variable with mean  $\mu$  and covariance matrix  $\Sigma$ , the mentioned probability can be computed as

$$\text{Prob}(\mathbf{a}^\top \mathbf{x} \leq b) = \frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{b - \mathbf{a}^\top \mu}{\sqrt{2\mathbf{a}^\top \Sigma \mathbf{a}}}\right), \quad (8)$$

where  $\text{erf}(\cdot)$  is the standard error function.

*Lemma 2:* Given  $\mathbf{x} \sim \mathcal{N}(\mu, \Sigma)$  and  $\delta \in (0, 0.5)$ , the probabilistic constraint can be transformed into a deterministic constraint:

$$\begin{aligned} \text{Prob}(\mathbf{a}^\top \mathbf{x} \leq b) &\leq \delta \\ \iff \mathbf{a}^\top \mu - b &\geq \text{erf}^{-1}(1 - 2\delta) \sqrt{2\mathbf{a}^\top \Sigma \mathbf{a}}. \end{aligned} \quad (9)$$

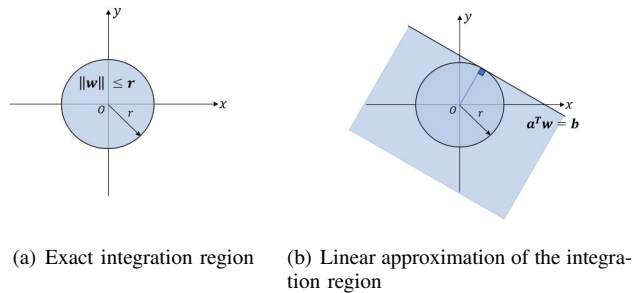
With *Lemma 1* and *Lemma 2*, the chance constraints in Eq. 2 and Eq. 5 can be transformed into deterministic ones that are easier to compute.

In the sensing or communication danger zone, the chance constraint computes multivariate integration across a disk. We linearly approximate the integration area to compute a conservative upper-bound of the probability. Such an approximation is illustrated in Fig. 1. As in Fig. 1(b), the boundary is approximated using a half-space formed by a line  $\mathbf{a}^\top \mathbf{w} = b$  that is tangent to the circle.

For a sensing danger zone with its hostile sensor centered at  $\mathbf{x}_{S_l} \sim \mathcal{N}(\mu_{S_l}, \Sigma_{S_l})$ , we can linearize it as

$$\text{Prob}(\mathbf{x}_i \in S_l) \leq \text{Prob}(\mathbf{a}_{i,S_l}^\top \mathbf{w}_{i,S_l} \leq r_l) \leq \epsilon_1. \quad (10)$$

In other words, we require the approximated probability to be bounded by  $\epsilon_1$ , ensuring a safety distance between robots and sensing danger zones. Here,  $\mathbf{a}_{i,S_l} = (\mu_{S_l} - \mathbf{x}_i) / \|\mu_{S_l} - \mathbf{x}_i\|$  is a unit vector. The chance constraint in Eq. 10 has the same



**Fig. 1:** Illustration of linearizing the integration region, to compute an upper-bound of the multivariate integral in Eq. 2 and Eq. 5. Fig. 1(a) shows the original circular integration region, while the half space in Fig. 1(b) is the approximation. The line  $\mathbf{a}^\top \mathbf{w} = b$  is tangent to the circle.

form as the one in *lemma 1* and *lemma 2*, thus is equivalent to the following deterministic constraint

$$\mathbf{a}_{i,S_l}^\top (\mu_{S_l} - \mathbf{x}_i) - r_l \geq \text{erf}^{-1}(1 - 2\epsilon_1) \sqrt{2\mathbf{a}_{i,S_l}^\top \Sigma_{S_l} \mathbf{a}_{i,S_l}}. \quad (11)$$

For simplicity of notation, we denote this as  $g_{S_l}(\mathbf{x}_i) \geq 0$ . We apply the same linearization to the chance constraints of communication danger zones. Consider the jamming station at  $\mathbf{x}_{C_k}$  that follows the Gaussian distribution  $\mathcal{N}(\mu_{C_k}, \Sigma_{C_k})$ . The probability in Eq. 6 is approximated as

$$\text{Prob}(a < \delta_2 c^*) \leq \text{Prob}(\mathbf{a}_{i,C_k}^\top \mathbf{v}_{i,C_k} \leq \delta_2 c^*) \leq 1 - \epsilon_2, \quad (12)$$

where  $\mathbf{a}_{i,C_k} = (\mu_{C_k} - \mathbf{x}_i) / \|\mu_{C_k} - \mathbf{x}_i\|$ . By *lemma 1* and *lemma 2*, it can be further transformed into a deterministic constraint

$$\mathbf{a}_{i,C_k}^\top (\mu_{C_k} - \mathbf{x}_i) - \delta_2 c^* \geq \text{erf}^{-1}(2\epsilon_2 - 1) \sqrt{2\mathbf{a}_{i,C_k}^\top \Sigma_{C_k} \mathbf{a}_{i,C_k}}. \quad (13)$$

We write this equation as  $h_{C_k}(\mathbf{x}_i) \geq 0$  for simplicity. With the approximation technique, our optimization program becomes:

$$\min_{\mathbf{u}_{i,t}, \mathbf{x}_{i,t+1}, \forall i} w_1 \cdot \text{Tr}(\mathbf{P}_{t+1}) + w_2 \cdot \sum_{i=1}^m \|\mathbf{u}_{i,t}\| \quad (14a)$$

$$\text{s.t.} \quad \mathbf{x}_{i,t+1} = \mathbf{f}_i(\mathbf{x}_{i,t}, \mathbf{u}_{i,t}), \forall i \in [m], \quad (14b)$$

$$g_{S_l}(\mathbf{x}_i) \geq 0, \forall i \in [m], \forall l \in [p], \quad (14c)$$

$$h_{C_k}(\mathbf{x}_i) \geq 0, \forall i \in [m], \forall k \in [q]. \quad (14d)$$

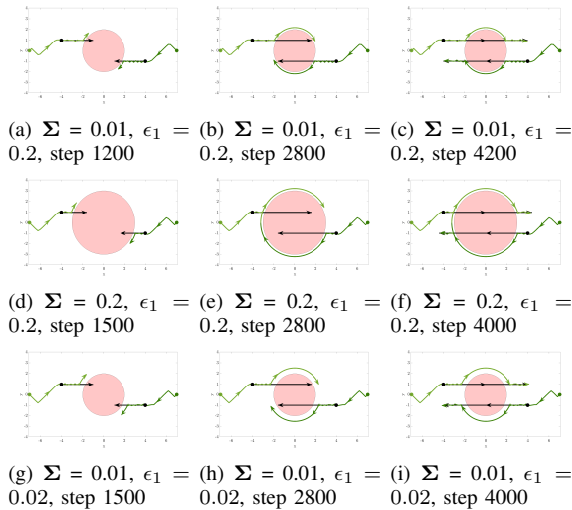
We use a solver named Forces Pro [7, 8] to solve program 14 and obtain control inputs for all the robots.

## IV. RESULTS

We evaluate the performance of our proposed method under the setting where (i) there exist sensing danger zones; and (ii) there exist communication danger zones.

### A. Sensing danger zones

We consider the scenario where two robots are tasked to track two targets in an environment with a sensing danger zone, with different covariance matrices  $\Sigma$  and confidence levels  $\epsilon_1$  of the dangerous zones. The qualitative results of three sets of parameter combinations are evaluated, and the trajectories of both robots and targets are shown in Fig. 2. We



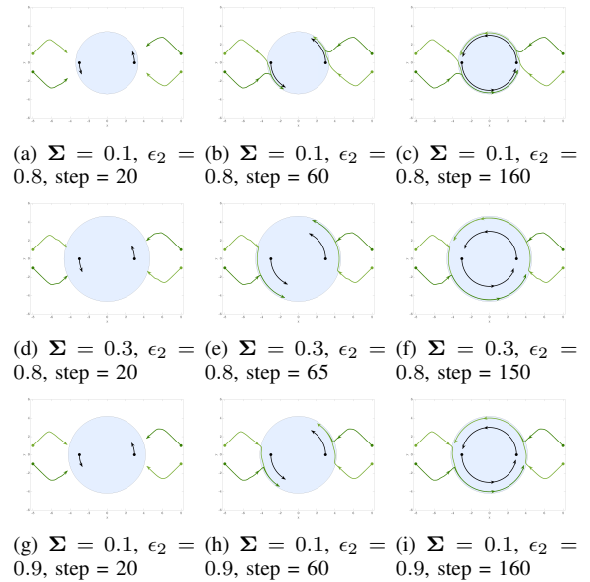
**Fig. 2:** The snapshots of trajectories when there is a sensing danger zone (pink) with three different parameter combinations. Each row corresponds to one pair of  $\Sigma$  and  $\epsilon_1$ , with three subfigures taken at different time steps. We use light and dark green to represent the trajectory of the two robots. The targets (black) are moving straight into the danger zone. The radius of circle qualitatively reflects the uncertainty of its central hostile sensor, e.g.,  $\Sigma = 0.2$  in the second row is higher, so we use larger disk to reflect it.

observed that the variation in parameters distinctly influences robots' trajectories when they are in proximity to sensing danger zones. The subfigures in row 1 and row 2 illustrate that increasing the uncertainty of danger zone forces the robots to be more conservative and to keep a larger distance from the sensing danger zone. This is consistent with our intuition that if the robots are more uncertain about where the hostile sensor is, they will be more discreet in their action and make a larger detour to avoid being detected. Moreover, from row 1 and row 3, decreasing the value of  $\epsilon_1$  imposes stricter requirements on robots, which prohibits them from entering the sensing danger zone. In this case, the robots also keep a larger distance away from the zone.

### B. Communication danger zones

We show comparative results with a communication danger zone to illustrate how the communication danger zone influences robots' behavior. We run three sets of experiments with different values for the uncertainty of the jammer  $\Sigma$  and confidence level  $\epsilon_2$ , and compare the trajectories of robots. We let four robots track two targets. We show the trajectories of robots and targets together in Fig. 3, where each row corresponds to one experiment and three subfigures in the same row show the tracking process under that setting.

Under all three parameter settings, the robots can successfully track and follow the targets. They divide themselves into 2-vs-2 sub-teams and each sub-team follows one target for better team-level tracking performance. This coincides with the characteristics of the Signal-to-Jamming ratio that the robots prefer to stay together and away from the jammer to prevent jamming. From row 1 and row 2, it is observed that increasing the uncertainty of the position of the jammer induces the robots to stay further away from the jammer.



**Fig. 3:** The snapshots of trajectories when there exists a communication danger zone (blue) with three different parameter combinations. Each row corresponds to one parameter setup, and three sub-figures in the same row show the tracking process under that setting. The robots' trajectories are plotted in different shades of green to distinguish these four robots. The trajectories of two targets (black) are in circles with a counter-clockwise direction.

This aligns with our intuition that since the jammer position is more uncertain, it is a safer strategy for robots to keep a larger distance from the jammer to protect the inter-robot communication from being interrupted. The comparison of row 3 and row 1 shows that increasing the confidence level  $\epsilon_2$  leads to a stricter risk requirement. The robots need to track targets from a larger safe gap. In other words, they need to be more confident that their communication channel will not be jammed.

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